Technology Company ROI And NPV
Technology Companies - Part III

Gary Schurman MBE, CFA

January 2017

Economic theory states that a company will make the investment in a new product (or product line) if the product's return on investment (ROI) is greater than the company's cost of capital. For technology companies that invest heavily in research and development (R&D), the return on investment using the company's GAAP financial statements will be incorrect because R&D is expensed as incurred and not capitalized. Our goal is to develop a model to calculate the return on investment for a technology company. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

The table below presents Intel's (Ticker: INTC) balance sheet and income statement at and for the twelve months ended December 31, 2020 (Dollars are in millions)...

Balance Sheet

Description	Balance
Cash and securities	31,239
Fixed assets (net)	56,584
Intangible assets	35,997
Other assets	29,271
Total assets	**153,091**
NIB liabilities	35,652
IB debt	36,401
Equity capital	81,038
Total financing	**153,091**

Income Statement

Description	Balance
Operating revenue	77,867
Operating expense	-40,435
R&D expense	-13,556
Non-operating income	1,202
Pre-tax income	**25,078**
Income tax expense	-4,179
Net income	**20,899**

Rates and Ratios

Capital assets ($)	85,855
Annualized revenue ($)	77,867
Net operating income ($)	37,432
Ratio cap assets to revenue	1.1000
Pre-tax revenue margin	0.4807
R&D expense to revenue	0.1741
Income tax rate	0.1666

Assume that weighted-average revenue life is 3 years, a new product introduced end of month 6, new product's annualized revenue at product introduction is $10 billion, after-tax cost of capital is 12.50%.

Table 1: Model Parameters

Symbol	Description	Value
R_t^s	Annualized revenue at time t on product brought to market at time s	$10,000
β	Weighted average revenue life in years	3.00
ϕ	Ratio of on-balance sheet capital assets to annualized revenue	1.1026
θ	Pre-tax revenue margin	0.4807
ω	Ratio of R&D expense to revenue	0.1741
κ	Cost of capital	0.1250
α	Income tax rate	0.1666

Question 1: What is Intel's return on investment using GAAP numbers?
Question 2: What is the total investment required to bring this product to market?
Question 3: What is the new product's net income and net cash flow in year one?
Question 4: What is the new product's pro-forma annualized return on investment?
Question 5: What is the new product's net present value?

Notes: Capital assets are defined as total assets minus cash and securities and intangible assets. Pre-tax net operating income excludes research and development expenditures and non-operating items. Revene margin is defined as the ratio of operating revenue minus operating expense to operating revenue.

Operating Revenue

We will define the variables s, t, m and n to be time in years. The relationship between these time variables are...

$$s \le t \text{ ...and... } s \le m \le n \tag{1}$$

We will define the variable R_t^s to be annualized revenue at time t on a product brought to market at time s, and the variable λ to be the rate of techological obsolesence (i.e. the revenue decay rate). The equation for annualized revenue is... [1]

$$R_t^s = R_s^s \,\mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \tag{2}$$

We will define the variable $R_{m,n}^s$ to be cumulative revenue recognized over the time interval $[m,n]$ on a product brought to market at time s. Using Equation (2) above, the equation for cumulative product revenue is...

$$R_{m,n}^s = \int\limits_m^n R_t^s\,\delta t = R_s^s \int\limits_m^n \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\}\delta t \tag{3}$$

Using Appendix Equation (40) below, the solution to Equation (3) above is...

$$R_{m,n}^s = R_s^s \left(\mathrm{Exp}\left\{ -\lambda\,(m-s)\right\} - \mathrm{Exp}\left\{ -\lambda\,(n-s)\right\}\right)\lambda^{-1} \tag{4}$$

We will define the variable β to be the product's weighted average revenue life. The equation for the rate of technological obsolesence as a function of weighted average revenue life is... [1]

$$\lambda = \frac{1}{\beta} \tag{5}$$

Revenue Margin

We will define the variable M_t^s to be annualized after-tax operating revenue margin at time t on a product brought to market at time s and the variable θ to be revenue margin as defined above. Using Equation (2) above, the equation for annualized after-tax revenue margin is...

$$M_t^s = \theta\,(1-\alpha)\,R_t^s = \theta\,(1-\alpha)\,R_s^s\,\mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \tag{6}$$

We will define the variable $M_{m,n}^s$ to be cumulative after-tax revenue margin recognized over the time interval $[m,n]$ on a product brought to market at time s. Using Equation (6) above, the equation for cumulative after-tax revenue margin over the time interval $[m,n]$ is...

$$M_{m,n}^s = \int\limits_m^n M_t^s\,\delta t = \theta\,(1-\alpha)\,R_s^s \int\limits_m^n \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\}\delta t \tag{7}$$

Using Appendix Equation (40) below, the solution to Equation (7) above is...

$$M_{m,n}^s = -\theta\,(1-\alpha)\,\lambda^{-1} R_s^s \left(\mathrm{Exp}\left\{ -\lambda\,(n-s)\right\} - \mathrm{Exp}\left\{ -\lambda\,(m-s)\right\}\right) \tag{8}$$

Investment in On-Balance Sheet Capital Assets

We will define on-balance sheet investment (i.e. capital assets) to be total GAAP assets minus intangible assets, cash and securities, and other non-operating items. We will define the variable A_t^s to be on-balance sheet investment at time t on a product brought to market at time s, and the variable ϕ to be the target ratio of on-balance sheet investment to annualized revenue. Using Equation (2) above, the equation for on-balance sheet investment is...

$$A_t^s = \phi\, R_t^s = \phi\, R_s^s\, \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \tag{9}$$

Using Equation (9) above, on-balance sheet investment required to bring the new product to market at time s is...

$$A_s^s = \phi\, R_s^s \tag{10}$$

The derivative of Equation (9) above with respect to time is...

$$\frac{\delta A_t^s}{\delta t} = -\lambda\,\phi\, R_s^s\, \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \text{ ...such that... } \delta A_t^s = -\lambda\,\phi\, R_s^s\, \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \delta t \tag{11}$$

We will define the variable $A_{m,n}^s$ to be cumulative return of capital investment over the time interval $[m,n]$ on a product brought to market at time s. Using Equation (11) above, the equation for the return of capital investment over the time interval $[m,n]$ is...

$$A_{m,n}^s = \int\limits_m^n \delta A_t^s = \lambda\,\phi\, R_s^s \int\limits_m^n \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \delta t \tag{12}$$

Using Appendix Equation (40) below the solution to Equation (12) above is...

$$A_{m,n}^s = \phi\, R_s^s \left(\mathrm{Exp}\left\{ -\lambda\,(n-s)\right\} - \mathrm{Exp}\left\{ -\lambda\,(m-s)\right\}\right) \tag{13}$$

Investment in Off-Balance Sheet Capital Assets

Off-balance sheet assets, which are comprised of capitalized research and development expenditures (after-tax), are development-related in that these assets represent past expenditures to develop a commercially viable product. We will define the variable D_t^s to be the unamortized balance of capitalized research and development expenditures at time t on a product brought to market at time s, the variable ω to be the dollar investment in research and development today that is required to generate one dollar of revenue in the future, and the variable α to be the income tax rate. Using Equation (2) above, the equation for unamortized after-tax product-related research and development expenditures is... [2]

$$D_t^s = \omega\,(1-\alpha)\,\lambda^{-1}\, R_t^s = \omega\,(1-\alpha)\,\lambda^{-1}\, R_s^s\, \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \tag{14}$$

Using Equation (14) above, off-balance sheet investment required to bring the new product to market at time s is...

$$D_s^s = \omega\,(1-\alpha)\,\lambda^{-1}\, R_s^s \tag{15}$$

The derivative of Equation (14) above with respect to time is...

$$\frac{\delta D_t^s}{\delta t} = -\omega\,(1-\alpha)\, R_s^s\, \mathrm{Exp}\left\{ \lambda\,(t-s)\right\} \text{ ...such that... } \delta D_t^s = -\omega\,(1-\alpha)\, R_s^s\, \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \delta t \tag{16}$$

We will define the variable $D_{m,n}^s$ to be the cumulative change in off-balance sheet investment (i.e. amortization of after-tax capitalized R&D) over the time interval $[m,n]$ on a product brought to market at time s. Using Equation (16) above, the equation for the cumulative change in off-balance sheet investment, which in this case is a non-cash expense, over the time interval $[m,n]$ is...

$$D_{m,n}^s = \int\limits_m^n \delta D_t^s = \omega\,(1-\alpha)\, R_s^s \int\limits_m^n \mathrm{Exp}\left\{ -\lambda\,(t-s)\right\} \delta t \tag{17}$$

Using Appendix Equation (40) below, the solution to Equation (17) above is...

$$D_{m,n}^s = \omega\,(1-\alpha)\, R_s^s \left(\mathrm{Exp}\left\{ -\lambda\,(n-s)\right\} - \mathrm{Exp}\left\{ -\lambda\,(m-s)\right\}\right) \lambda^{-1} \tag{18}$$

Net Income

We will define the variable $N_{m,n}^s$ to be cumulative net income over the time interval $[m,n]$ on a product brought to market at time $s \leq m$. Net income is defined as after-tax revenue margin minus amortization of capitalized after-tax research and development expenditures. Using Equations (7) and (17) above, the equation for cumulative net income is...

$$N_{m,n}^s = \int\limits_m^n M_t^s \, \delta t + \int\limits_m^n \delta D_t^s \tag{19}$$

Using Equations (8) and (18) above the solution to Equation (19) above is...

$$N_{m,n}^s = (\omega - \theta)\,(1-\alpha)\,\lambda^{-1} R_s^s \bigg(\text{Exp}\bigg\{ -\lambda\,(n-s) \bigg\} - \text{Exp}\bigg\{ -\lambda\,(m-s) \bigg\} \bigg) \tag{20}$$

Net Cash Flow

We will define the variable $C_{m,n}^s$ to be cumulative after-tax cash flow over the time interval $[m,n]$ on a product brought to market at time s. Using Equations (7) and (12) above, the equation for cash flow is...

$$C_{m,n}^s = \int\limits_m^n M_t^s \, \delta t - \int\limits_m^n \delta A_t^s = (\theta\,(1-\alpha) + \lambda\,\phi)\, R_s^s \int\limits_m^n \text{Exp}\bigg\{ -\lambda\,(t-s) \bigg\} \, \delta t \tag{21}$$

Using Appendix Equation (40) below the solution to Equation (21) above is...

$$C_{m,n}^s = -\lambda^{-1} (\theta\,(1-\alpha) + \lambda\,\phi)\, R_s^s \bigg(\text{Exp}\bigg\{ -\lambda\,(n-s) \bigg\} - \text{Exp}\bigg\{ -\lambda\,(m-s) \bigg\} \bigg) \tag{22}$$

Note that we can combine terms and rewrite Equation (22) above as...

$$C_{m,n}^s = (\theta\,(1-\alpha)\,\lambda^{-1} + \phi)\, R_s^s \bigg(\text{Exp}\bigg\{ -\lambda\,(m-s) \bigg\} - \text{Exp}\bigg\{ -\lambda\,(n-s) \bigg\} \bigg) \tag{23}$$

Net Present Value (NPV)

We will define the variable $V_{m,n}^s$ to be the present value at time s of net cash flow over the time interval $[m,n]$ on a product brought to market at time s, and the variable κ to be the risk-adjusted discount rate. Using Equation (21) above, the equation for the present value of net cash flow is...

$$\begin{aligned} V_{m,n}^s &= \int\limits_m^n M_t^s \, \text{Exp}\bigg\{ -\kappa\,(n-s) \bigg\} \, \delta t - \int\limits_m^n \delta A_t^s \, \text{Exp}\bigg\{ -\kappa\,(n-s) \bigg\} \\ &= (\theta\,(1-\alpha) + \lambda\,\phi)\, R_s^s \int\limits_m^n \text{Exp}\bigg\{ -\lambda\,(t-s) \bigg\} \, \text{Exp}\bigg\{ -\kappa\,(t-s) \bigg\} \, \delta t \\ &= (\theta\,(1-\alpha) + \lambda\,\phi)\, R_s^s \int\limits_m^n \text{Exp}\bigg\{ -(\lambda+\kappa)\,(t-s) \bigg\} \, \delta t \end{aligned} \tag{24}$$

Using Appendix Equation (40) below, the solution to Equation (24) above is...

$$\begin{aligned} V_{m,n}^s &= -(\lambda+\kappa)^{-1} (\theta\,(1-\alpha) + \lambda\,\phi)\, R_s^s \bigg(\text{Exp}\bigg\{ -(\lambda+\kappa)\,(n-s) \bigg\} - \text{Exp}\bigg\{ -(\lambda+\kappa)\,(m-s) \bigg\} \bigg) \\ &= (\lambda+\kappa)^{-1} (\theta\,(1-\alpha) + \phi\,\lambda)\, R_s^s \bigg(\text{Exp}\bigg\{ -(\lambda+\kappa)\,(m-s) \bigg\} - \text{Exp}\bigg\{ -(\lambda+\kappa)\,(n-s) \bigg\} \bigg) \end{aligned} \tag{25}$$

If we redefine the time interval $[m, n]$ to be $[s, \infty]$ then present value Equation (25) above becomes...

$$\begin{aligned} V^s_{s,\infty} &= (\lambda+\kappa)^{-1}(\theta\,(1-\alpha)+\phi\,\lambda)\,R^s_s\Bigg(\mathrm{Exp}\Bigg\{-(\lambda+\kappa)\,(s-s)\Bigg\}-\mathrm{Exp}\Bigg\{-(\lambda+\kappa)\,(\infty-s)\Bigg\}\Bigg) \\ &= (\lambda+\kappa)^{-1}(\theta\,(1-\alpha)+\phi\,\lambda)\,R^s_s\Bigg(1-0\Bigg) \\ &= (\lambda+\kappa)^{-1}(\theta\,(1-\alpha)+\phi\,\lambda)\,R^s_s \end{aligned} \tag{26}$$

Return on Investment (ROI)

ROI is the discount rate that makes the present value (present value of future net cash flow minus the initial investment at time zero) equal to zero. Using Equations (9), (14) and (26) above, the equation that we want to solve is...

$$V^s_{s,\infty} = A^s_s + D^s_s \tag{27}$$

The solution to Equation (27) above where κ is the return on investment is...

$$\begin{aligned} (\lambda+\kappa)^{-1}(\theta\,(1-\alpha)+\phi\,\lambda)\,R^s_s &= \phi\,R^s_s+\omega\,(1-\alpha)\,\lambda^{-1}\,R^s_s \\ \theta\,(1-\alpha)+\phi\,\lambda &= (\lambda+\kappa)\,(\epsilon+\phi+\omega\,(1-\alpha)\,\lambda^{-1}) \\ \kappa &= (\theta\,(1-\alpha)+\phi\,\lambda)/(\phi+\omega\,(1-\alpha)\,\lambda^{-1})-\lambda \end{aligned} \tag{28}$$

The return on investment in Equation (28) above is a continuous-time return. The equation for the discrete-time annualized rate is...

$$\text{if... Continuous-time ROI} = \kappa \text{ ...then... Discrete-time ROI} = \mathrm{Exp}\Bigg\{\kappa\Bigg\}-1 \tag{29}$$

The Answer To Our Hypothetical Problem

Using Equation (5) above the equation for λ is...

$$\lambda = \frac{1}{\beta} = \frac{1}{3.00} = 0.3333 \tag{30}$$

Question 1: What is Intel's return on investment using GAAP numbers?

$$\text{GAAP ROI} = \frac{\text{Net income excluding non-op items}}{\text{Capital assets}} = \frac{20,899-1,202\times(1-0.1666)}{85,855} = 23.18\% \tag{31}$$

Question 2: What is the total investment required to bring this product to market?

Using Equation (10) and (15) above and the data in Table 1 above, the answer to the question is...

$$\text{On-BS} = A^6_6 = 10,000\times1.1026 = 11,026 \text{ ...and... Off-BS} = D^6_6 = 10,000\times0.1741\times(1-0.1666) = 4,352 \tag{32}$$

Using Equation (32) above, the answer to the question is...

$$\text{Total investment} = 11,026+4,352 = 15,378 \tag{33}$$

Question 3: What is the new product's net income and net cash flow in year one?

Using Equations (20) and (30) above and the data in table 1 above, cumulative net income is...

$$\begin{aligned} N^6_{6,12} &= (0.1741-0.4807)\times(1-0.1666)\times0.3333^{-1}\times10,000\times\Bigg(\mathrm{Exp}\Bigg\{-0.3333\times(12-6)\Bigg\} \\ &\quad -\mathrm{Exp}\Bigg\{-0.3333\times(6-6)\Bigg\}\Bigg) = 1,177 \end{aligned} \tag{34}$$

Using Equations (22) and (30) above and the data in table 1 above, cumulative net income is...

$$C^6_{6,12} = (0.4807 \times (1 - 0.1666) \times 0.3333^{-1} + 1.126) \times 10,000 \times \Bigg(\text{Exp}\Bigg\{ - 0.3333 \times (6 - 6)\Bigg\} - \text{Exp}\Bigg\{ - 0.3333 \times (12 - 6)\Bigg\}\Bigg) = 3,538 \tag{35}$$

Question 4: What is the new product's pro-forma annualized return on investment?

Using Equation (28) and (30) above and the data in table 1 above, the continuous-time return on investment is...

$$\kappa = (0.4807 \times (1 - 0.1666) + 1.1000 \times 0.3333)/(1.1000 + 0.1741 \times (1 - 0.1666) \times 0.3333^{-1}) - 0.3333 = 0.1662 \tag{36}$$

Using Equations (29) and (32) above, the answer to the question is...

$$\text{Discrete-time annualized ROI} = \text{Exp}\Bigg\{0.1662\Bigg\} - 1 = 18.08\% \tag{37}$$

Question 5: What is the new product's net present value?

Using Equation (26) and (30) above and the data in table 1 above, the present value of the new product's cash flow discounted at the risk-adjusted cost of capital is...

$$V^6_6 = (0.3333 + 0.1250)^{-1} \times (0.4807 \times (1 - 0.1666) + 1.1026 \times 0.3333) \times 10,000 = 16,759 \tag{38}$$

Using Equations (33) and (38) above, the answer to the question is...

$$\text{Product NPV} = 16,759 - 15,378 = 1,381 \tag{39}$$

Appendix

A. The solution to the following integral is...

$$\int\limits_m^n \text{Exp}\Bigg\{ - \lambda\,(t - s)\Bigg\}\,\delta t = -\lambda^{-1}\text{Exp}\Bigg\{ - \lambda\,(t - s)\Bigg\}\Bigg[_m^n = -\lambda^{-1}\Bigg(\text{Exp}\Bigg\{ - \lambda\,(n - s)\Bigg\} - \text{Exp}\Bigg\{ - \lambda\,(m - s)\Bigg\}\Bigg) \tag{40}$$

B. Using Equation (40) above the solution to the following integral is...

$$\int\limits_s^\infty \text{Exp}\Bigg\{ - (\lambda + \kappa)\,(t - s)\Bigg\}\,\delta t = -(\lambda + \kappa)^{-1}\text{Exp}\Bigg\{ - (\lambda + \kappa)\,(t - s)\Bigg\}\Bigg[_s^\infty = -(\lambda + \kappa)^{-1}\Bigg(\text{Exp}\Bigg\{ - (\lambda + \kappa)\,\infty\Bigg\} - \text{Exp}\Bigg\{ - (\lambda + \kappa)\,(s - s)\Bigg\}\Bigg) = (\lambda + \kappa)^{-1} \tag{41}$$

References

[1] Gary Schurman, *Modeling Technology Product Revenue,* December, 2019.

[2] Gary Schurman, *Capitalized R$D Expenditures,* December, 2019.